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Study of Transition in a High-Disturbance Environment

R. D. McDaniel* and H. A. Hassan†
North Carolina State University, Raleigh, North Carolina 27695-7910

A theory is developed for a form of bypass transition that is induced by a high-disturbance environment and is used to study transitional flows at low speeds. It employs an approach similar to that developed for natural transition. In this approach, transitional flows are treated in a turbulencelike manner, and transition onset and extent are determined as part of the solution. It is shown that this form of bypass transition is a result of a receptivity mechanism, where the disturbance is characterized by a scale different from that of Tollmien–Schlichting waves. The theory is calibrated and validated by considering a range of low-speed flows, with high freestream turbulence intensities, in the presence and absence of pressure gradients, referred to as the T3 test cases. Excellent agreement with measurements is indicated. It is suggested that transition in one of the test cases, T3C4, is a result of the bursting of a laminar separation bubble.

Introduction

GUIDED by results of linear stability theory, Warren and Hassan[1–3] developed an approach that is capable of economically calculating laminar/transitional/turbulent flows using traditional Reynolds-averaged Navier–Stokes codes. In this approach, transitional flows are treated in a turbulencelike manner, with the transitional eddy viscosity deduced from the results of linear stability theory. As a result, transition onset, which may correspond to minimum skin friction, minimum heat transfer at the wall, etc., is calculated as part of the solution, thus eliminating the need to use stability codes or empirical correlations. The approach has been applied with great success to a variety of complex flows over a wide range of Mach numbers where transition was triggered by various mechanisms.[4–7] Because of reliance on linear stability theory, application of the approach is limited to those situations where turbulence is a result of natural transition.

The goal of the present investigation is to extend the approach of Warren and Hassan[1–3] to transitions induced by high-disturbance environment (HIDE). In such an environment, the intensity of the fluctuations in the freestream exceeds 1%. This is typical of flows that exist in noisy tunnels and turbomachines. This type of transition is one of many that were labeled by Markovin as bypass transition.[8] It covers a class of transitional flows where the linear Tollmien–Schlichting (T–S) mechanism is completely bypassed by a nonlinear mechanism (see Ref. 9). Two approaches have been used to study such flows: approaches based on traditional two-equation and/or stress turbulence models with and without an intermittency equation[10–12] and conditional Navier–Stokes equations coupled to an intermittency transfer equation (see Ref. 13). As was pointed out by Savill on a number of occasions (for example, see Ref. 10), occasional good agreement of the first approach with experiment is fortuitous. This is because turbulence models are developed without regard to the transitional mechanisms that lead to turbulence. On the other hand, transitional flows are triggered by various mechanisms, and models that describe such flows must reflect the mechanisms responsible for transition. This suggests that traditional low Reynolds number turbulent models that were developed to improve predictions of near-wall behavior cannot be expected to describe the evolution of the flow from laminar to transitional to turbulent along a surface. Improvement in prediction is enhanced when an intermittency equation is used.[12] This improvement is due, in part, to the fact that such an equation requires, in addition, an onset criterion, which can be obtained from experiment or empirical correlations. Although the second approach[13] is physically more appealing, it does not have the flexibility to incorporate other types of transitional flows, and it is difficult to incorporate into traditional computational fluid dynamics codes.

The present approach will employ the k–ζ transitional/turbulent model, and thus, it maintains both the conceptual and computational simplicity of the earlier work of Warren and Hassan.[1–3] The model will be calibrated and validated by examining all of the T3 test cases.[9] This set of experiments involves low-speed flows with zero and variable pressure gradients, upstream turbulence intensities ranging from 0.9 to 6.6%, and a range of Reynolds numbers.

Approach

The message that Morkovin tried to convey in Ref. 8 is that, bypass transition, of which HIDE is a subset, can not be described by linear stability theory. Furthermore, nonlinear theories have yet to be developed to address this type of transition. Because of this, an approach based on dimensional considerations will be employed.

There were a number of considerations involved in extending the work of Refs. 1–3 to transition induced by HIDE. The first dealt with receptivity. Is this form of bypass transition a result of entraining disturbances into the boundary layer thereby triggering perturbations that amplify, or is it a result of diffusion into the boundary layer? If we are dealing with a receptivity mechanism, can we describe transition induced by HIDE by replacing the T–S mechanisms by another strongly amplifying mechanism? In weighing these and other issues, we were guided by Reynolds's experiment in a pipe in which he showed that transition onset shifts with the upstream intensity of the incoming flow. This suggests that a receptivity mechanism is suited for this form of bypass transition. As a result, the resulting model can be obtained by a straightforward modification of the Warren and Hassan model.

To show how the modification was arrived at, a brief review of the model[1–3] will be presented. In this model, the turbulent viscosity μ_t is replaced by

$$(1-\Gamma)\mu_{\mathrm{nt}} + \Gamma\mu_t$$

where the subscript nt designates nonturbulent fluctuations and Γ is the intermittency. The eddy viscosity μ_{nt} is chosen as

$$\mu_{\mathrm{nt}} = c_\mu \rho k \tau_{\mathrm{nt}}, \qquad c_\mu = 0.09 \tag{1}$$

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*Research Assistant, Mechanical and Aerospace Engineering Department. Student Member AIAA.

†Professor, Mechanical and Aerospace Engineering Department. Associate Fellow AIAA.

where ρ is the density, k is the fluctuation kinetic energy per unit mass, and τ_{nt} is the timescale of the nonturbulent fluctuations. For transition resulting from T–S waves,

$$\tau_{\mathrm{nt}} = a/\omega, \qquad \omega\nu\big/U_e^2 = 0.48Re_x^{-0.65} \tag{2}$$

where a is a model constant that depends on the freestream intensity, ν is the kinematic viscosity, U_e is the edge velocity, Re_x is the Reynolds number, and ω is the frequency of the most amplified mode. The correlation indicated in Eq. (2) was derived in Ref. 5 using results obtained by Mack.[14] Similarly, the dissipation timescale in the k equation was chosen as[1−3]

$$1/\tau_k = (1-\Gamma)(1/\tau_{k,\mathrm{nt}}) + \Gamma(1/\tau_{k,t}) \tag{3}$$

where, for T–S waves

$$\frac{1}{\tau_{k,\mathrm{nt}}} = a\frac{\nu_{\mathrm{nt}}}{\nu}S, \qquad S^2 = S_{ij}S_{ij}, \qquad S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{4}$$

For the current form of bypass transition, Eqs. (2) and (4) were modified to

$$\tau_{\mathrm{nt}} = a_1\left(\nu\big/U_e^2\right)Re_x^{\alpha}$$

$$1/\tau_{k,\mathrm{nt}} = (a_2\nu_{\mathrm{nt}}/\nu + a_3\Gamma)S \tag{5}$$

where a_1, a_2, a_3, and α are model constants.

Although the approach can in principle be implemented with any one- or two-equation turbulence model, the k–ζ turbulence model of Robinson and Hassan[15] is employed because it is free of damping and wall functions and is coordinate independent. Furthermore, all model correlations are tensorially consistent and Galilean invariant. In addition, the model reproduces the correct growth rates of all free shear layers and is capable of predicting separated flows in the presence and absence of shocks.

Intermittency and Onset Prediction Criteria

The intermittency Γ is that developed by Dhawan and Narashima,[16] that is,

$$\Gamma(x) = 1 - \exp(-0.412\xi^2), \qquad \xi = \max(x - x_t, 0)/\lambda \tag{6}$$

where λ is the characteristic extent of the transition region and is determined from the correlation

$$Re_\lambda = 9.0Re_{xt}^{0.75} \tag{7}$$

with x_t being the location where turbulent spots first appear, or the onset of transition. In this work, x_t is determined as part of the solution.

When the pressure gradient is zero, transition onset is assumed to correspond to the location of minimum skin friction. When the pressure gradient is not zero, this criterion is not appropriate. Instead, we employ a variation of a criterion developed in Ref. 1 that describes the growth of the eddy viscosity of the nonturbulent fluctuations. This criterion assumes that onset corresponds to the point where

$$\nu_{\mathrm{nt}}/\nu = 1.0 \tag{8}$$

is first achieved.

Because of the way intermittency is implemented in this work, the transitional profiles are not a simple combination of purely laminar and purely turbulent profiles. When the results obtained are considered, the approach seems to compensate for the need for conditional averaging.

Model Constants

The model constants were arrived at by considering a couple of the T3 test cases. There are all kinds of waves, including T–S waves present in the flow. However, only certain types are amplified and eventually result in transition. Thus, the suggestion that the division between natural and this form of bypass transition is not as sharp as Morkovin suggested[10] is not supported. To prove this, our first attempt at modeling transition induced by HIDE was to assume that τ_{nt} scales like T–S waves, that is, Eq. (2) is valid. Although one can determine one set of constants that can work for one case, it became quickly clear that the strategy will not work for the remaining cases.

Although the results presented here are obtained using a modification of the compressible turbulent boundary-layer code developed by Harris and Blanchard,[17] the goal is to develop a model that is suited for implementation in Navier–Stokes solvers. Because in most wind tunnels the distribution of intensity in the test section is not provided, the development is based on the assumption that the average intensity is generally available. In this work, this quantity was determined by calculating the average intensity provided in the database. All calculations were started at 10 mm downstream from the leading edge. At that station, the kinetic energy k and the enstrophy ζ were obtained from the assumed edge ν_t/ν, which can be 10^{-1} or less, and the average intensity. Thus, no profiles of k or ζ were assumed. This corresponds to the freestream boundary-layer condition that is normally specified in Navier–Stokes codes.

All of the results to be presented assume

$$a_1 = 0.0012, \qquad a_2 = 0.0036, \qquad a_3 = 0.34$$

$$\alpha = 0.9448(1 \pm 0.02)Tu^{\frac{1}{8}} \tag{9}$$

where Tu is the average intensity.

Results and Discussion

The flow geometry used in the test consists of a flat plate with the external turbulence being generated by grids mounted upstream. The variable pressure gradient is imposed by modifying the shape of the opposite tunnel wall. The experimental data were obtained by Roach and Brierley[18] and are available on the Web site URL: http://vortex.mech.surrey.ac.uk.

To the best of our knowledge, none of the existing models were able to reproduce accurately all of the T3 test cases. Here, all eight cases are considered: the three involving zero pressure gradient ($T3A^-$, T3A, and T3B) and the five involving favorable/unfavorable pressure gradient (T3C1–T3C5). As is will be seen, the model successfully predicted all cases except T3C4.

As indicated earlier, the initial input consists of a constant value for k, based on the average intensity, and a constant value for ν_t/ν (a range of values $< 10^{-1}$ was considered), which provided the initial value for ζ. This is typical of approaches used in calculations involving Navier–Stokes solvers. The nature of the current model is such that calculated values of ζ are only used downstream of the onset location. As a result, the model is not sensitive to initial values of ν_t/ν that determine the initial ζ. This, however, is not the case for models that do not make use of an intermittency equation.

The grid used has 201 points in the direction normal to the wall and steps as low as 1 mm were used in the flow direction. Thus, all of the results presented here are grid independent.

Figures 1–3 compare predicted (——) and measured (□) skin-friction coefficients c_f for the test cases $T3A^-$, T3A, and T3B. A minimum skin-friction criterion is used to determine transition onset

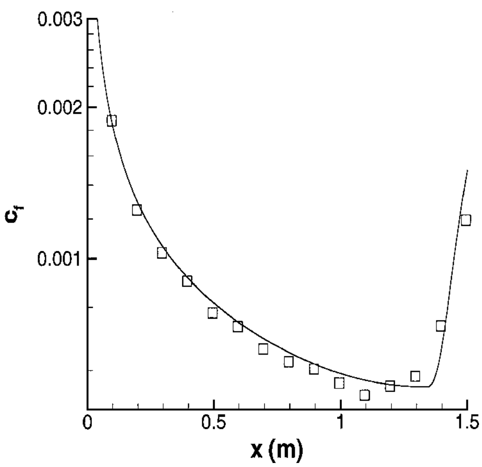

Fig. 1 Measured (□) and computed (——) skin friction, $T3A^-$.

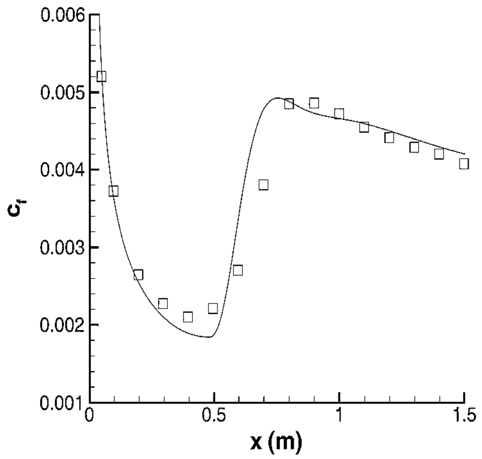

Fig. 2 Measured (□) and computed (——) skin friction, T3A.

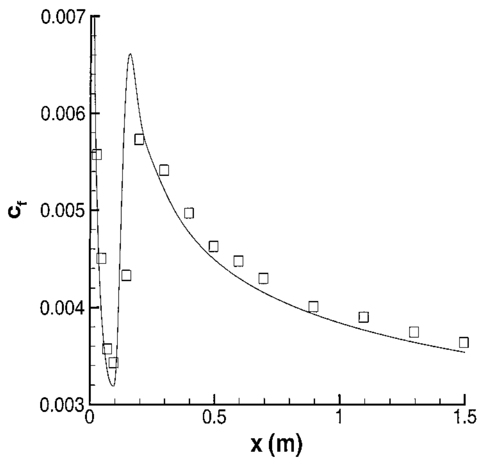

Fig. 3 Measured (□) and computed (——) skin friction, T3B.

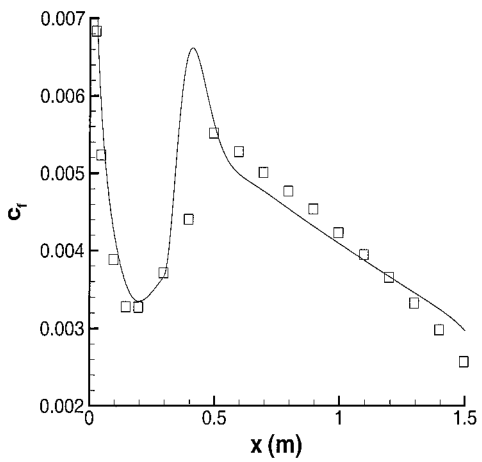

Fig. 4a Measured (□) and computed (——) skin friction, T3C1.

for these test cases. As is seen in Figs. 1–3, transition onset together with measured skin friction are well predicted. Some differences are noted in c_f in the transitional region. It is not exactly clear how the experimental values are arrived at and what the experimental error is.

The results where pressure gradient is not zero are discussed next. Because the Mach number is very small, the pressure coefficient c_p can be calculated from

$$c_p = 1 - (U_e/U_\infty)^2 \tag{10}$$

where U_e and U_∞ are the edge and freestream velocities. Note, however, that the values of c_p, U_e, and U_∞ provided in the database are not consistent with Eq. (10). An effective U_∞ was calculated for each case to ensure compliance of data with Eq. (10). Another problem of the data is the lack of detailed pressure measurements in regions where transition takes place and toward the end of the plate. A sixth-degree polynomial was used to fit the pressure data, and this was used to determine the pressure gradient.

The results for when the pressure gradient is different from zero are given in Figs. 4–8. Figures 4a–7a compare predictions of c_f with experiment for test cases T3C1–T3C3 and T3C5, whereas Figs. 4b–7b show experimentally provided edge velocity scaled with freestream velocities indicated in Table 1. Equation (8) is used to determine transition onset for these cases. As is seen from Figs. 4a–7a, good agreement is indicated. Most of the deviations take place in

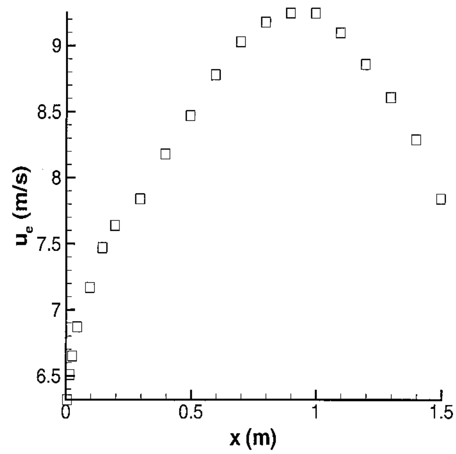

Fig. 4b Measured edge velocity, T3C1.

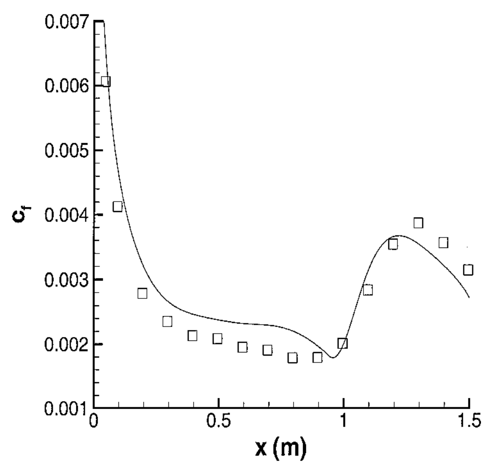

Fig. 5a Measured (□) and computed (——) skin friction, T3C2.

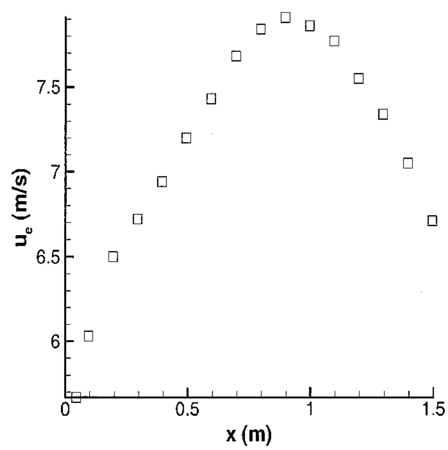

Fig. 5b Measured edge velocity, T3C2.

Table 1 Given and calculated quantities for T3 test cases

Case name	U_∞ given, m/s	U_∞ average, m/s	Tu_∞ given, %	Tu average, %	Pressure gradient
T3A$^-$	19.8	19.8	0.9	0.592	Zero
T3A	5.4	5.22	3.0	1.69	Zero
T3B	9.4	9.51	6.0	3.45	Zero
T3C1	5.9	5.75	6.6	2.74	Variable
T3C2	5.0	4.88	3.0	1.16	Variable
T3C3	3.7	3.62	3.0	1.09	Variable
T3C4	1.2	1.26	3.0	1.15	Variable
T3C5	8.4	8.24	3.0	1.20	Variable

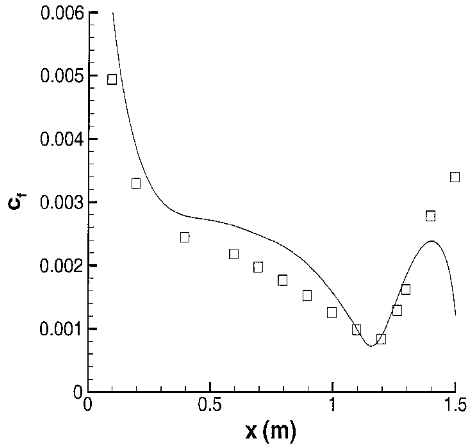

Fig. 6a Measured (□) and computed (——) skin friction, T3C3.

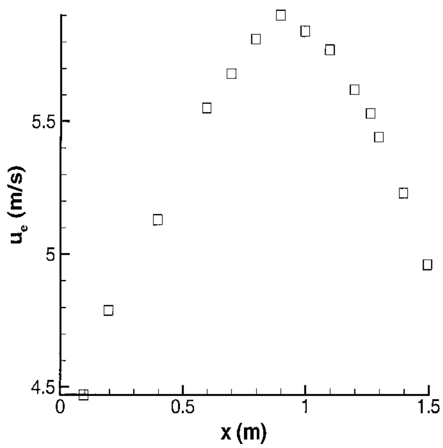

Fig. 6b Measured edge velocity, T3C3.

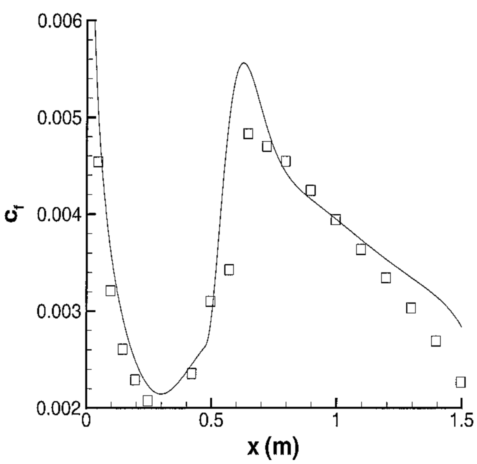

Fig. 7a Measured (□) and computed (——) skin friction, T3C5.

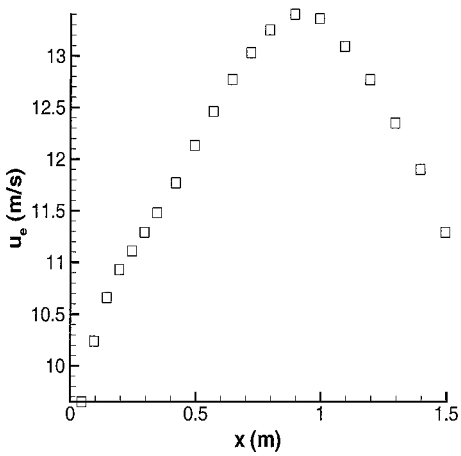

Fig. 7b Measured edge velocity, T3C5.

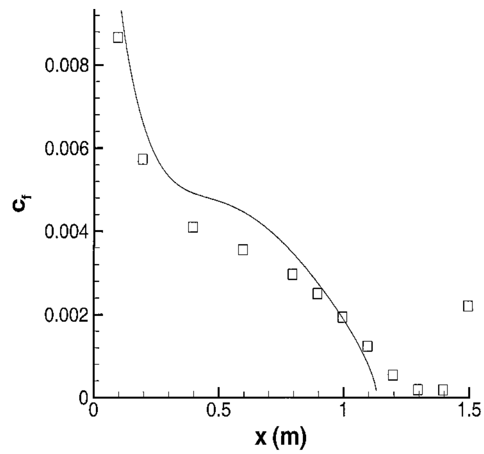

Fig. 8a Measured (□) and computed (——) skin friction, T3C4.

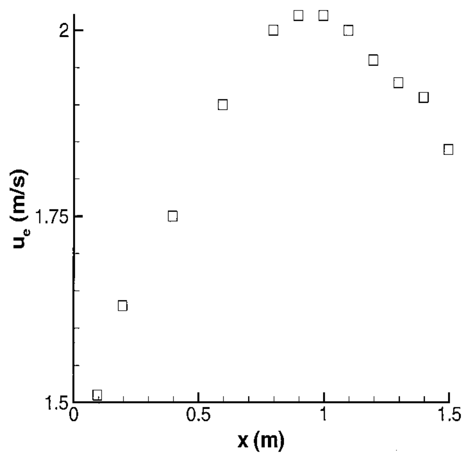

Fig. 8b Measured edge velocity, T3C4.

the transitional region. Other deviations may be a result of the fitted pressure data.

Figure 8a compares prediction of skin-friction coefficient with experiment from the T3C4 case whereas the experimentally measured edge velocity is given in Fig. 8b. As is seen from Fig. 8a, theory predicts separation before transition. As is seen from Fig. 8a, measured values at the last station is about 12 times higher than values at the previous two stations: 0.00202 compared to 0.000183 and 0.000187. Because of the low Reynolds number, there is a good possibility that transition in this case is a result of the burst of a laminar separation bubble.

Conclusions

It is shown that the framework developed for determining the onset and extent of transition developed for natural transition can be extended to transition induced by HIDE. The scale of the disturbance responsible for transition is, however, quite different from that of a T–S wave.

The results presented here are in good agreement with the T3 experiments. The only exception is that for T3C4. Present results suggest transition is a result of the bursting of a laminar separation bubble for this case.

With this development, the designer has another tool needed to calculate transitional flows resulting from transition induced by a HIDE, at a reasonable cost.

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